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Aspects combinatoires et algorithmiques des fonctions L d'Artin

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Les fonctions L d'Artin sont des objets centraux en théorie algébriques des nombres qui peuvent servir à l'étude des extensions non-abéliennes des corps de nombres et sont l'objet du programme de Langlands.

Nous avons implanté dans le système de calcul formel PARI/GP des algorithmes permettant le calcul des coefficients et des valeurs numériques des fonctions zeta de Dedekind, et des fonctions L de Hecke et d'Artin.

En première partie nous rappelons les définitions et les propriétés et conjectures sur les fonctions L d'Artin.

En seconde partie, nous montrons comment le théorème de Brauer et les propriétés de fonctorialité des fonctions L d'Artin peuvent être utilisés pour accélérer les algorithmes de calculs.

Pour conclure, nous montrons comment ces algorithmes permettent de donner des exemples numériques pour le théorème de Langlands-Tunnel et la correspondance de Deligne-Serre avec les formes modulaires de poids 1, que nous pouvons calculer grâce à un programme dû à Henri Cohen et Karim Belabas qui est disponible dans PARI/GP.

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Several identities of generalized Stirling numbers and degenerate Bernoulli polynomials and poly-Cauchy polynomials

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As generalizations of one simple formula for Harmonic numbers, we give several identities of generalized Stirling numbers and degenerate Bernoulli polynomials. We present some expressions of values at nonnegative integers of poly-Cauchy polynomials as finite sums involving Stirling numbers. We also give expressions of Stirling numbers as finite sums involving values of poly-Cauchy polynomials, as well as generating series for values of poly-Cauchy polynomials at nonnegative integers.

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On Gneralized Hopfian Abelian Group in the Category of Algebraically Compact Abelian Group

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An abelian group A is called Hopfian if for any surjective endomorphism f of A then f is an automorphism of A . In this paper We will characterize the Gneralized Hopfian abelian group in the category of Algebraically Compact abelian group. We know that the p -component of Gneralized Hopfian torsion abelian group is also Gneralized Hopfian. After we construct a Gneralized Hopfian abelian group but its the p -component of A isn't Gneralized Hopfian abelian group.

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On the Sato-Tate conjecture

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The recent proof of the Sato-Tate Conjecture is one of the breakthrough results in mathematics recently by Barnet-Lamb, Geraghty, Harris, Shepherd-Barron and Taylor. The Sato-Tate Conjecture is a statement about the statistical distribution of certain sequences of numbers. Let $k \geq 2$ and $f = \sum_{n \geq 1} a(n)q^n$ be a normalised cuspidal Hecke eigenform of weight $2k$ for $\Gamma_0(N)$ without complex multiplication. Then the Sato-Tate Conjecture says that the numbers $\frac{a(p)}{2p^{(k-1)/2}}$ are equidistributed in $[-1, 1]$ with respect to a certain measure when p runs through the primes not dividing N . By Modularity Theorem, it is also possible to state the conjecture for elliptic curves. Therefore, we can state the conjecture in terms of Frobenius angles. In this case, the Sato-Tate Conjecture tells these angles are distributed according to the function where is the Frobenius angle with . In this talk, we will consider the Sato-Tate Conjecture for modular forms with its history and consequences.

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Subfields of the 3-Hilbert class field of $\mathbb{Q}(\sqrt[3]{p}, \zeta_3)$

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Let $k = \mathbb{Q}(\sqrt[3]{p}, \zeta_3)$ the normal closure of the pure cubic field $\mathbb{Q}(\sqrt[3]{p})$, where p is a prime number such that $p \equiv 1 \pmod{9}$, and let $k_3^{(1)}$ be the 3-Hilbert class field of k . The 3-component of the class group of k is isomorphic to $\mathbb{Z}/9\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$. By the aid of class field theory, we determine all unramified sub-extensions of $k_3^{(1)}/k$.

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Dèveloppement en fractions continues de l'opèrateur de Heinz

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Soient A et B deux matrices définies positives, α un nombre réel tel que $0 \leq \alpha \leq 1$. L'opérateur de Heinz de A et B est définie par:

$$H_{\alpha}(A, B) = A^{\frac{1}{2}} \cdot \frac{\left(A^{-\frac{1}{2}}BA^{-\frac{1}{2}}\right)^{\alpha} + \left(A^{-\frac{1}{2}}BA^{-\frac{1}{2}}\right)^{1-\alpha}}{2} \cdot A^{\frac{1}{2}} \quad 0 < \alpha < 1.$$

Le calcul direct de cet opérateur s'avère difficile par l'apparition des exposants rationnels de matrices.

La motivation principale de ce travail est de dépasser ces difficultés et de présenter une méthode pratique et efficace pour ce calcul en utilisant les fractions continues matricielles.

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Idéaux réduits d'un corps cubique pur cas monogène

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Soit $K = \mathbb{Q}(\sqrt[3]{m})$ Un corps cubique pur, où m est un entier sans facteur cubique et et soit α une racine cubique de m appartenant à K . si $m \not\equiv \pm 1 \pmod{9}$, on dit que K est de première espèce, si en outre m est sans facteur carrée, alors l'anneau des entiers de K est $O_K = [1, \alpha, \alpha^2]$. Dans ce papier nous allons déterminer les idéaux réduits d'un tel corps.

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Rhythms of Arabic words and Fibonacci words

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The rhythms of Arabic words, as they were studied by *Al Khalil Al Farahidi* (أَلْخَلِيلُ الْفَرَاهِدي), are the sequences of movement (حركة) and silence (سكون), which take into account only the letters uttered in a word. *Al Khalil* considers the letter (حرف) and its movement (حركة) as a single phonetic unit. This definition is different from that using consonant and vowel. The study of the Arabic metric (علم العروض) allowed us to identify the rhythms of Arabic words and, subsequently, the rhythms of speech. We will demonstrate that these rhythms are variants of Fibonacci words.

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On the normality of a monogenic algebra

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Let $(R, \pi R, k)$ a discrete valuation ring, $\mathfrak{p} = \pi R$ the prime ideal of R , and k its residue field. Let K the quotient field of R , L a finite extension of K , O_L the integral closure of R in L , $\alpha \in O_L$ a primitive element of L , $P = \text{Irr}(\alpha, K) \in R[X]$. the purpose of this paper is to construct a ring that contains $R[X]/(P)$ and contained in O_L in the case where P does not satisfy the Dedekind criterion, that is to say that the R -algebra $R[x]/(P)$ is not normal.

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RSA trapdoor

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We could factor a RSA number of 33403 size, in a time of 1367,942 second. A twin prime is a prime number that has a prime gap of two, in other words, differs from another prime number by two, for example the twin prime pair (5, 3). In this work we define a new notion: " r -prime number of degree k " and we give a new RSA trap-door one-way. This notion generalized a twin prime numbers because the twin prime numbers are 2-prime numbers of degree 1.

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Classification automatique des textures par les Réseaux de Neurones Artificiels

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La texture joue un rôle très important dans l'identification et l'extraction des informations thématiques contenues dans l'image. L'analyse des textures est un vaste champ dont l'objectif est d'identifier la nature d'une texture, soit via des algorithmes de classification, soit via des algorithmes synthétiques visant à la création d'une texture, visuellement similaire à la texture d'origine.

Notre objectif est de faire une reconnaissance des différents types de tumeurs en travaillant sur des images médicales et de décrire une nouvelle approche de la reconnaissance automatique des textures dans les images numériques en utilisant des réseaux de neurones artificiels [RNA]. Pour cette raison, on a essayé dans un premier temps de tester sur la base de données Brodatz.

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On P -2-Bézout Rings

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In this paper, we introduce a generalization of the well-known notion of a " P -Bézout" ring and a "2-Bézout" ring, which we call a " P -2-Bézout" ring. We establish the transfer of this notion and the notion of "2-Bézout" ring to trivial ring extensions. We conclude with a brief discussion of the scope and limits of our results.

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Characterization of the Automorphisms of any Free Cyclic Module over Integral Factorial having the Extension Property

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Let A be an integral factorial ring, if M is a module over A and $\alpha \in \text{Aut}_A(M)$. We say that α satisfies the Extension Property, if pour tout monomorphism $\sigma : M \rightarrow N$ there existe $\hat{\alpha}$ such that the following diagram

$$\begin{array}{ccc} M & \xrightarrow{\sigma} & N \\ \alpha \downarrow & & \downarrow \hat{\alpha} \\ M & \xrightarrow{\sigma} & N \end{array}$$

We will show that every automorphism of divisible satisfies the Extension Property, but this result isn't true in general case. After we will give some properties of the automorphisms having the Extension Property. Finally we will Characterize the automorphism of Free Cyclic Module having the Extension Property

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Nil-clean property in amalgamated algebras along an ideal

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Let $f : A \longrightarrow B$ be a ring homomorphism and let J be an ideal of B . In this Talk, we give a characterization for the amalgamation of A with B along J with respect to f (denoted $A \bowtie^f J$) (introduced and studied by D'Anna, Finocchiaro, and Fontana in [6] and [7]) to be nil-clean.

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Variant of Schnorr zero-knowledge protocol

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Identification is an important issue in public key cryptography. It is used in various situations, such as authorization to access to a server, digital signatures, exchange of communication between a customer and bank. The security of identification protocol is based on difficult mathematical questions, like discrete logarithm, factoring and computing square root modulo a large composite number. In 1989 Schnorr proposed an identification scheme based on the discrete logarithm problem. In this work we propose an identification protocol inspired by Schnorr scheme. We study its security and complexity.

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On the rank of the 2-class group of pure quartic number field $\mathbb{Q}(\sqrt[4]{pd^2})$ where $p \equiv 5 \pmod{8}$

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Let d be positive square-free integers, $p \equiv 5 \pmod{8}$ be prime a with $(p, d) = 1$. Our goal is to compute the rank of the 2-class group of pure quartic number field $\mathbb{Q}(\sqrt[4]{pd^2})$ by the ambiguous class formula .

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Characterization of Extrema of a Pseudoconvex Functions in Terms of Proximal Subdifferential

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In this paper, we give necessary and sufficient optimality conditions for a point to be an extremum of a pseudoconvex function over a convex set. Our principal tool is the proximal subdifferential.

Keywords: Nonconvex calculus, pseudoconvex functions, limiting subdifferential, proximal subdifferential, normal cone.

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Capitulation of the 2-ideal classes of $K=k\left(\sqrt{-p\varepsilon\sqrt{l}}\right)$ of type (2, 2, 2)

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Let $p \equiv 3 \pmod{4}$ and $l \equiv 5 \pmod{8}$ be different primes such that $\left(\frac{p}{l}\right) = 1$ and $\left(\frac{p}{l}\right)_4 = 1$. Put $k = \mathbb{Q}(\sqrt{l})$ and denote by ε its fundamental unit. Let $K = k\left(\sqrt[4]{-p\varepsilon\sqrt{l}}\right)$, $K_2^{(1)}$ be its Hilbert 2-class field and $K_2^{(2)}$ be the Hilbert 2-class field of $K_2^{(1)}$. The 2-class group $C_{K,2}$ of K is of type (2, 2, 2). Our goal, in this communication, is to determine the structure of the metabelian Galois group $G = \text{Gal}(K_2^{(2)}/K)$ thus to study the capitulation of the 2-ideal classes of K in all its unramified abelian extensions within $K_2^{(1)}$.

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On Drazin-Ruston elements of a Banach algebra

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We introduce the class of Drazin-Ruston elements with respect to a homomorphism between two Banach algebras. In this work, we extend some of the Ruston theory by among other things, developing the Ruston and almost Ruston elements, and spectra relative to an arbitrary homomorphism. In addition we provide a number of application and generalize certain well-known results.

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A note on finite products of fields

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Let R be a commutative ring. Suppose that R is zero-dimensional, it would be interesting to check whether R contains a finite product of fields. Recently many papers have studied Artinian subrings of a commutative ring and direct limit of finite product of fields ([4, 5, 6, 8]). Recall that Artinian rings form an important class of zero-dimensional rings. Moreover, an Artinian ring has only finitely many idempotent elements. Essentially, the characterization of the set of Artinian subrings of a commutative ring is known (see [6]). In this talk we are interested in the Artinian overring of pair of rings, that means, we are looking for intermediate Artinian rings between R and T , where R is a subring of a von Neumann regular ring T .

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Lattice Based Cryptography

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On assiste actuellement à une perpétuelle évolution de l'informatique Quantique ; Et la préoccupation du monde de la sécurité est de concevoir des cryptosystèmes capable de résister à des éventuelles attaques de l'ordinateur Quantique. La cryptographie " Lattice-Based-Cryptography " ; est jugée capable de résister à des telles attaques, et assurera la relève des cryptosystèmes classiques comme RSA , ECDH, etc. qui ont atteints leurs limites. Dans cet exposé ; qui représente un Etat d'Art sur la cryptographie basée sur les réseaux euclidiens, on représentera un rappel des bases théoriques, ainsi que quelques problèmes sur lesquelles est construite la cryptographie Lattices ; puis on décrira le système NTRU qui est considéré comme un pilier de la cryptographie Lattices, Enfin on conclut avec les axes de la recherche sur cette nouvelle méthode de cryptographie.

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On Iwasawa theory of Rubin-Stark units and narrow class groups

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Let K be a totally real number field of degree r . Let K_∞ denote the cyclotomic \mathbb{Z}_2 -extension of K and let L_∞ be a finite extension of K_∞ , abelian over K . The goal of this talk is to compare the characteristic ideal of the χ -quotient of the projective limit of the narrow class groups to the χ -quotient of the projective limit of the r -th exterior power of totally positive units modulo a subgroup of Rubin-Stark units, for some \mathbb{Q}_2 -irreducible characters χ of $\text{Gal}(L_\infty/K_\infty)$.

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Crypto Système à Clé Publique de McEliece basé sur les Produits Codes matrices

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Les applications des produits matrice codes ont également une utilisation pratique en théorie des codes, dans la création des produits matrice codes en cryptosystème PMC est un cryptosystème de McEliece basés sur les produits matrice codes PMC, une famille de codes de distance minimale. Dans cette communication, on essaye d'introduire un crypto système à clef publique utilisant des codes correcteurs d'erreurs (c'est un système deux en un). Le système étudié est le crypto système de McEliece utilisant le produit des codes et des matrices.

Nous allons dans cette communication : faire une cryptographie en proposant une amélioration de l'attaque de décodage, précisément celui du décodage par ensemble d'information sur les codes binaires et ternaires.

Finalement on va proposer une application du cryptosystème de Mc.Elièce en proposant une utilisation des : " Matrix-Product Codes ".

Références

Sur la \mathbb{Z}_2 -extension cyclotomique de certains corps quadratiques réels

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Soient p_1, p_2 et q des premiers différents tels que $p_1 \equiv p_2 \equiv -q \equiv 1 \pmod{4}$, et q_1, q_2 et q_3 des premiers différents tels que $q_1 \equiv q_2 \equiv q_3 \equiv -1 \pmod{4}$, désignons par $A(\mathbb{k}), A(\mathbb{k}')$ et $A(\mathbb{k}_1)$ les 2-groupes de classes respectifs des corps $\mathbb{k} = \mathbb{Q}(\sqrt{d}), \mathbb{k}' = \mathbb{Q}(\sqrt{2d})$ et $\mathbb{k}_1 = \mathbb{Q}(\sqrt{2}, \sqrt{d})$, où $d = p_1 p_2 q$ ou $d = q_1 q_2 q_3$. Dans ce papier, on s'intéresse à déterminer tous les corps \mathbb{k} tels que $A(\mathbb{k}) \simeq A(\mathbb{k}_1)$ ou $A(\mathbb{k}') \simeq A(\mathbb{k}_1)$.

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Quasi-Cyclic Codes

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Quasi-cyclic codes over a finite commutative ring are viewed as cyclic codes over a noncommutative ring of matrices over a finite commutative ring. The study of these codes permits to generalize some known results about quasi-cyclic codes over a finite fields and to propose a construction of some quasi-cyclic codes.

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How to generate secure elliptic curves for efficient cryptographic applications

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Nowadays, elliptic curves are widely used in many cryptographic applications to secure information exchanged or stored in public networks [1, 2]. In practice, generating good elliptic curves in a cryptographic context is a very difficult task. The first curves were standardized by the NIST (in 2000) [3], but new concerns appeared since then.

In this paper, we discuss the mathematical proprieties of elliptic curves over finite fields for cryptographic applications [4], and we propose a generation process of new curves using PARI/GP system [5].

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Electronic voting on elliptic curves and security

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The elections require large human and financial resources, electronic voting offers a cheaper method, more secure and it is difficult to buy the votes. In this work we will talk about cryptography on elliptic curves over fields and over ring, as well as these applications to electronic voting, the motivation is to use secure electronic voting systems based on very difficult cryptographic problems.

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Some characterizations of Jordan homomorphisms on Banach algebras

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In this paper, we characterize Jordan homomorphisms on Banach algebras by product preserving conditions of generalized Drazin invertibility. More explicit forms of such maps are given in the context of the algebra of all bounded linear operators on a complex Banach space.

An additive map ϕ between unital Banach algebras is said to be *Jordan homomorphism* if $\phi(a^2) = \phi(a)^2$ for all a , or equivalently $\phi(ab + ba) = \phi(a)\phi(b) + \phi(b)\phi(a)$ for all a, b ; and is said to be *unital* if $\phi(1) = 1$.

It is well-known that every unital Jordan homomorphism ϕ between unital Banach algebras preserves strongly invertibility, that is $\phi(a^{-1}) = \phi(a)^{-1}$ for every invertible element a , see [6]. Moreover, a well-known formulation of Hua's theorem from [5] asserts that every bijective additive unital map on a field K preserving strongly invertibility is either an automorphism or an anti-automorphism. This result was later improved in [2] where it was shown that any bijective additive map $\phi : K \rightarrow K$ such that $\phi(a)\phi(a^{-1}) = \phi(b)\phi(b^{-1})$ for every non-zero elements $a, b \in K$ is of the form $\phi = \phi(1)\psi$ where $\psi : K \rightarrow K$ is either an automorphism or an anti-automorphism and $\phi(1)$ commutes with every element of K . Hua's theorem was later extended to Banach algebras in [1].

In this paper, our aim consists in giving analogue results for generalized Drazin invertibility and group invertibility.

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Introduction sur les corps quartiques cycliques réels

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L'exposé est une introduction aux corps quartiques cycliques réels. Je vais présenter les caractéristiques et les formes de ces corps pour aboutir à la forme convenable à l'étude de groupe de classes et qui assurée l'existence d'une base integral relative.

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